On the Capabilities of CP for Numerical Program Analysis

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Outline

- **BMC** (Bounded Model Checking)
  - Goal : *Finding counter-examples violating an assertion*
  - State of the art Methods → *SAT /SMT Solvers*

- **Program analysis**
  - Goal: *Get rid of false alarms*
  - State of the art Methods → *abstract interpretation*

- **Fault localization**
  - Goal: *locations of potentially faulty statements*
  - State of the art Methods → *MaxSat*

Bounded Model Checking

- **Context**: programs with ***numeric operations*** over integer or floating point numbers

- **Goal** : *Finding counter-examples violating an assertion*

Bounded Model Checking framework

Models → finite automates, labelled transition systems

Properties:

- Safety → something bad should not happen
- Liveness → something good should happen

Bound *k* → look only for counter examples made of *k* states

Bounded Model Checking framework (cont.)

% set of states: S, initial states: I, transition relation: T

% bad states B reachable from I via T?

bounded_model_checker$_{forward}$(I,T,B,k)

SC =∅; SN =I; n=1

while $S_C \neq S_N$ **and** n<k **do**

If $B \cap S_N \neq \emptyset$

then return *"found error trace to bad states"*;

else $S_C = S_N$; $S_N = S_C \cup T(SC)$; $n = n + 1$;

done

return *"no bad state reachable"*;

SAT/SMT - Based BMC framework

1 The *program is unwound k* times

2 The unwound (and simplified) *program* and the *negation* of the property are translated into *a big propositional formula* φ

φ is satisfiable iff there exists a counterexample of depth less than k

SAT solvers solvers have a *"**Global** view"*

Numerical expressions → **Boolean abstraction**

→ **Spurious solutions**

Critical issue: ***relevant minimum conflict sets*** to ***limit backtracks***

CP-Based BMC framework

1 The ***program is unwound k*** times

2 The unwound (and simplified) program in ***SSA/DSA form*** and the ***negation property*** are translated ***on the fly*** into ***constraint system Cs***

Cs is satisfiable for a full path iff there exists a counterexample of depth less than k

Various solvers and ***strategies*** can be used

To explore only a limited part of the search space, ***efficient pruning is a critical issue***

CP-Based BMC: CPBPV, a depth first strategy

CPBPV:

- Translate ***precondition*** (if exists) and ***property*** to check into a set of constraints
- ***Explore each branch Bi*** of the program and translate statements of branch *Bi* into a set of constraints
  - If for each branch *Bi,* the generated ***CSP is inconsistent*** , then the ***program is conform*** with its specification
  - If for some branch *Bi* the generated ***CSP has a solution,*** then this solution is a ***counterexample*** → exhibits ***a non-conformity***

Inconsistencies are detected at each node of the control flow graph

CP-Based BMC: DPVS, a Dynamic Backjumping Strategy

Start from the post-condition and jump to the first locations where the variables of the post-condition are assigned

Essential observation:

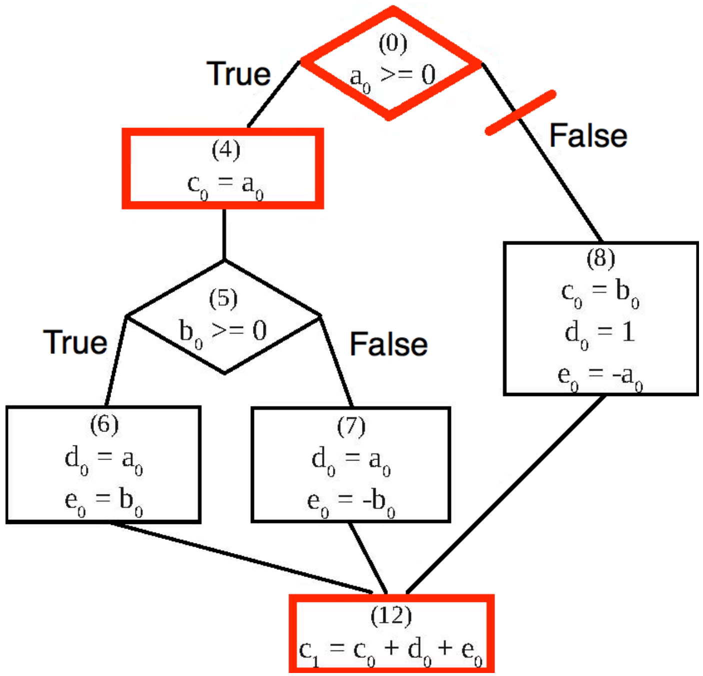
When the program is in an SSA-like form, CFG does not have to be explored in a top down (or bottom up) way

→ ***compatible blocks can just be collected in a non-deterministic way***

Why does it pay off ?

- **Enforces the constraints** on the domains of the selected variables
- **Detects inconsistencies earlier**

CP-Based BMC: DPVS, example



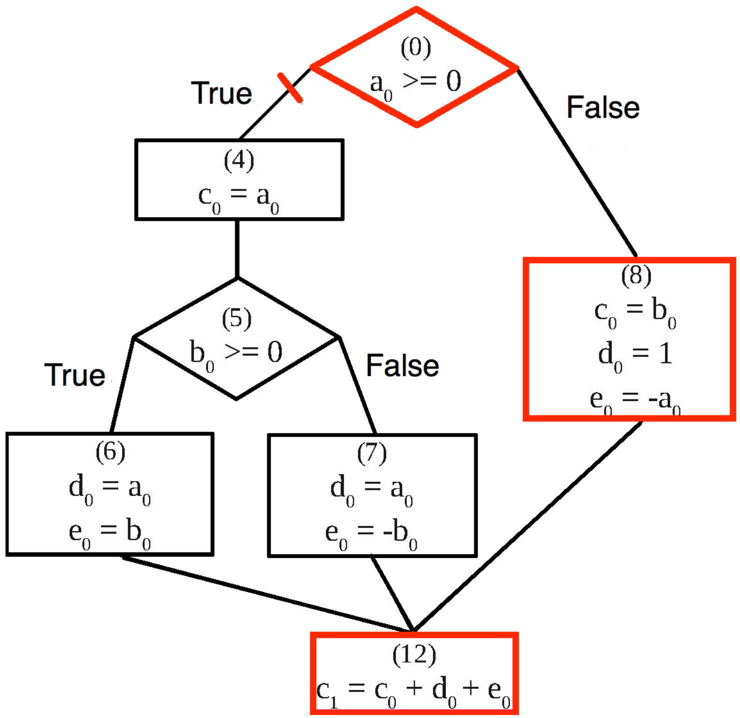

```
void foo(int a, int b)
int c, d, e, f;
if(a>=0) {
        if (a<10)  {f=b-1;}
        else {f=b-a;}
        c=a;
        if (b>=0) {d=a; e=b}
        else {d=a; e=-b;}    }
else {
        c=b; d=1; e=-a;
        if (a>b) {f=b+e+a;}
        else {f=e*a-b;}  }
c = c+ d + e;
assert(c>=d+e); // property p1
assert(f>=-b*e); // property p2
```

To prove property p_1, select node (12), then select node (4)

→ the condition in node (0) must be true

$S=\{c_1 < d_0+e_0 \wedge c_1 = c_0+d_0+e_0 \wedge c_0 = a_0 \wedge a_0 \geq_0\} = \{a_0 < 0 \wedge a_0 \geq 0\}$ ... **inconsistent**

CP-Based BMC: DPVS, example (cont.)



void foo(**int** a, **int** b)
int c, d, e, f;
if(a>=0) {
    ~~**if** (a<10) {f=b-1;}~~
    ~~**else** {f=b-a;}~~
    c=a;
    if (b>=0) {d=a; e=b}
    else {d=a; e=-b;} }
else {
    c=b; d=1; e=-a;
    ~~**if** (a>b) {f=b+e+a;}~~
    ~~**else** {f=e*a-b;} }~~
c = c+ d + e;
assert(c>=d+e); // property p_1
assert(f>=-b*e); // property p_2

Select node (8) → condition in node (0) must be false:

$S = \{c1 < d_0 + e_0 \wedge c1 = c_0 + d_0 + e_0 \wedge c_0 = b_0 \wedge a_0 < 0 \wedge d_0 = 1 \wedge e_0 = -a_0\}$

$\quad = \{a_0 < 0 \wedge b_0 < 0\}$

Solution $\{a0 = -1, b0 = -1\}$

CP-Based BMC: Static versus Dynamic Strategies

Two benchmarks:

- **Flasher Manager**, industrial application
- **Binary Search**

Bench	DPVS	CPBPV
FM 5	0.5	1.24
FM 100	15.95	> **600**
FM 200	**22.65**	> 600
BS 8	35	0.2
BS 16	> **600**	**1.14**

→ ***Pruning is a critical issue***

CP-Based program analysis

- **Context**:
  - **Embedded Systems** (Anti-lock Braking System controller, ...) rely more and more on floating-point computations
  - C language is widely used for such applications (often C code generated from a Simulink model)

    Floats $\longrightarrow$ a source of **execution errors**

- **Goal:** ***Get rid of false alarms*** *(generated by abstract interpretation tools)*

Problems with floating-point numbers

Rounding: ***Counter-intuitive properties***

- Arithmetic operators are neither associative nor distributive
- Reasoning with ***absorption*** and ***cancellation***

Examples (in simple precision, binary representation):

- Absorption: $10^7 + 0.5 = 10^7$
- Cancellation: $((1 - 10^{-7}) - 1) * 10^7 = -1.192...(\neq -1)$
- $(10000001-10^7)+0.5 \neq 10000001-(10^7+0.5)$
- $0.1=(0.000110011001100...)$

Problems with floating-point numbers (cont.)

Programs are run on the floats **but**:

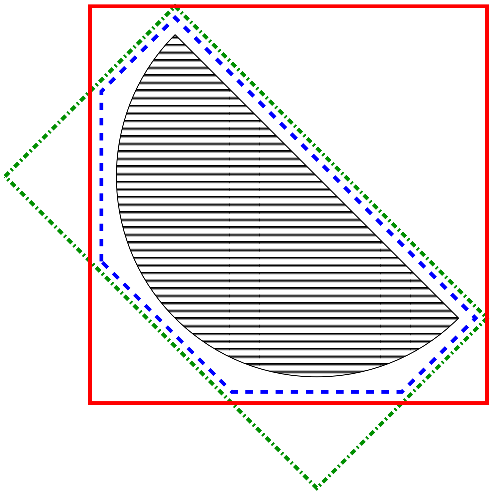
- **Specification, properties** of programs

  $\longrightarrow$ Users are **reasoning with real numbers**

- **Programs** are often written with the semantics of real numbers "in mind"
- **Differences** between computations over real numbers and computations over the floats

  $\longrightarrow$ ***Execution problems on programs with floats***

Abstract Interpretation



Goal: static detection of execution errors

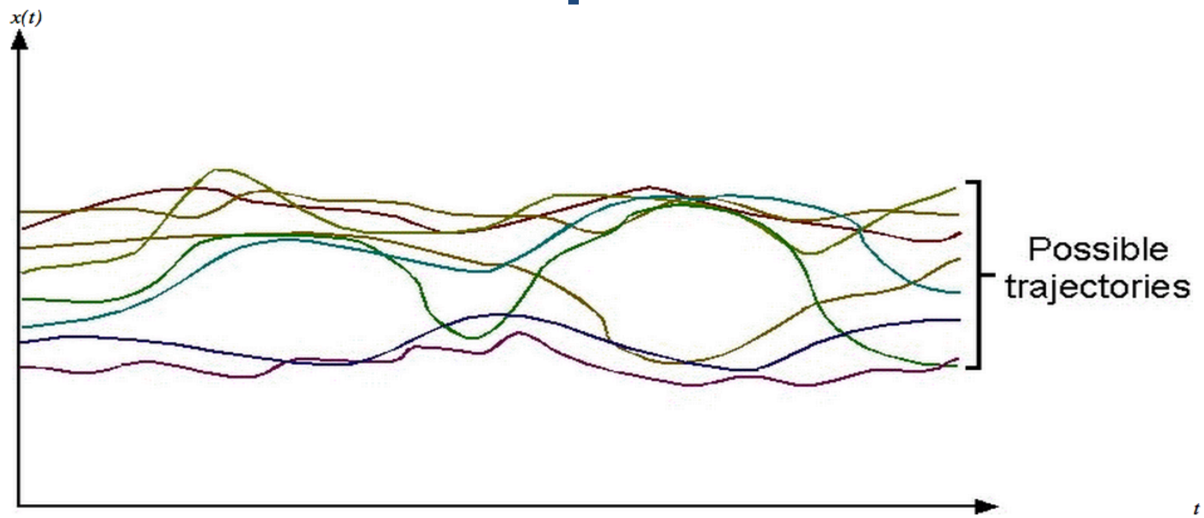
→ Approximations of computations over floats and of computations over the real numbers

Intervals, **zonotopes**, **polyhedra**...

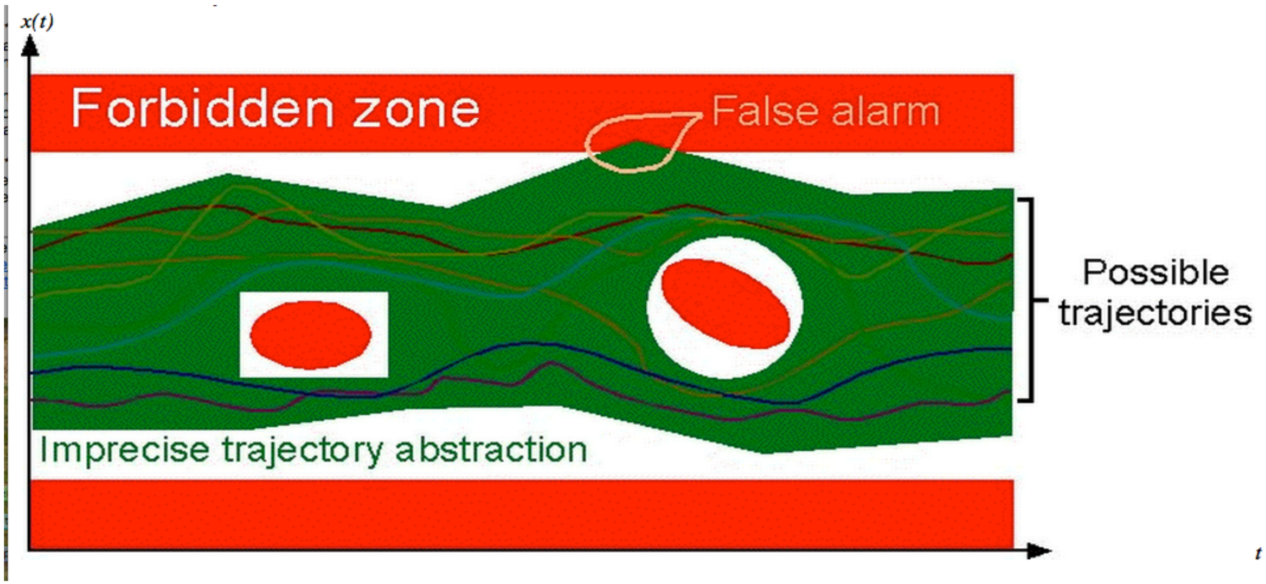
Zonotopes: convex polytopes with a central symmetry (sets of affine forms)

- \+ **Good trade-off** between performance and precision
- – **Not very accurate** for nonlinear expressions and on very common program constructs such as conditionals

Limits of Abstract Interpretation: false alarms



Courtesy
to Patick Cousot



AI versus CP

Abstract Interpretation: ***good scalability*** for estimating rounding errors but *over-approximation*

→ ***false alarms***

→ *totally* ***inappropriate behaviours*** *of a program may be dreaded but the developer does not know whether these behaviours will actually occur !*

Constraint Programming :

Good precision (strong refutation capabilities, finding counter examples) but ***lack of scalability***

rAICP: Combining AI and CP (cont.)

Successive exploration and merging steps

- Use of ***AI to compute a first approximation*** of the values of variables at a program node where two branches join

- Building a constraint system for each branch between two join nodes in the CFG of the program and use of ***CP local consistencies to shrink the domains*** computed by AI

rAICP: example

```
1 /* Pre-condition:   f,g ∈ [-10,10]  */
2 float foo(float f, float g) {
3   float x, y, z;
4 
5   x = f + 2 * g;
6 
7   if (x <= 0) {
8     y = g;
9   }
10  else {
11    y = -g;
12  }
13 
14  if (y >= 0) {
15    z = 10*y;
16  }
17  else {
18    z = -y;
19  }
20 
21  return z;
22 }
```

On floats and reals, foo → z= [0,50]

Fluctuat → z= [0,100]

Merge points of foo: lines 13 and 21

lines 1 →13:

Fluctuat → $f,g,y \in [-10,10]$, $x \in [-10, 0]$

FPCS (path 1, "then" branch) :

$C = \{x = f + 2*g \wedge x \leq 0 \wedge y = g \wedge -10 \leq f \wedge f \leq 10 \wedge -10 \leq g \wedge g \leq 10 \wedge -10 \leq y \wedge y \leq 10 \wedge -10 \leq x \wedge x \leq 0\}$

$\rightarrow g,y \in [-10,5]$

lines 14 → 22:

Fluctuat → $z \in [0,50]$

rAICP: Filtering techniques

FPCS: ***solver over floating-point constraints*** combining ***interval propagation*** with explicit ***search***

- **Correct** solver over the floats : ***no solutions are lost***
- Based on ***2B-consistency*** and ***3B-consistency***

  Projection functions for floats:

  - *Direct projections*: straightforward adaptation of interval arithmetic
  - Inverse projections: less intuitive, more complex (e.g., might need a larger format than the system variables)
- ***Handling of rounding modes***, nonlinear expressions and the usual mathematical functions (trigonometric. . .)

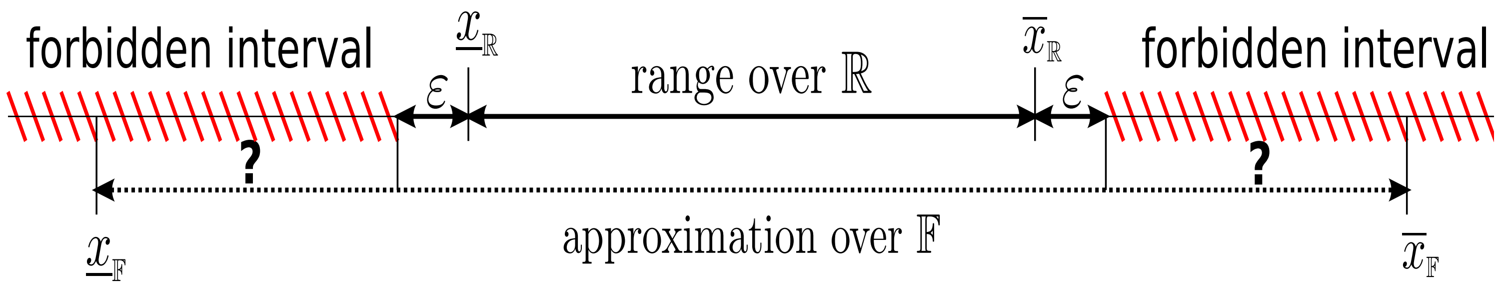
Experiments: eliminating false alarms

CDFL: Program analyser for proving the absence of runtime errors in program with floating-point computations based on *Conflict-Driven Learning*

	rAICP	**Fluctuat**	**CDFL**
False alarms	**0**	**11**	**0**
Total time	40.55s	**18.33 s**	**208.99** s

Computed on the 55 benchs from CDFL paper (TACAS'12, *D'Silva, Leopold Haller, Daniel Kroening, Michael Tautschnig*)

Generating Test Cases inside Suspicious Intervals



- **Suspicious intervals** for *x* : $[\underline{\mathbf{x}}_F , \underline{\mathbf{x}}_R - \varepsilon]$ or $[\mathbf{x}_R + \varepsilon, \mathbf{x}_F]$
- Tolerance specified by the user : ε
- **Question:** *Can the program hit a forbidden zone over the floating-point numbers?*

Proposed approach : CPBPV_FP

"Forward" propagation

Computing the suspicious interval of *x*

→ approximate the domain of x over the reals by

→ approximate domain of x over the floats by $[\underline{\mathbf{x}}_F, \mathbf{x}_F]$

"Backward " propagation

Computing test-cases inside a suspicious interval of *x*

→ Solving a bounded-model checking problem with domain of x restricted to $[\underline{\mathbf{x}}_F, \underline{\mathbf{x}}_R - \varepsilon]$ or $[\mathbf{x}_R + \varepsilon, \mathbf{x}_F]$

CPBPV_FP : CP based BMC for floats

Outputs :

- **A test case**

  $\longrightarrow$ *P* ***can produce*** a suspicious value for *x*
- **A proof that no test case exists**

  $\longrightarrow$ the suspicious interval ***can be removed***

  Only the case if the loops in P cannot be unfolded beyond the bound k
- **An inconclusive answer**

  $\longrightarrow$ *P* ***may produce*** a suspicious value

no test case could be generated
but the loops in P could be unfolded beyond the bound k

FPCS Search Strategies

- **std:** standard *prune & bisection-based search*
- **fpc:** domain of selected variable is split in *5 intervals*
  - 3 degenerated intervals: the smallest float ***l***, the largest float ***r***, and the mid-point ***m***
  - intervals ***]l, m[*** and ***]m, r[***
- **fp3s:** domain of selected variable is split in 3 degenerated intervals: the smallest float ***l***, the largest float ***r***, and the mid-point ***m***

Experiments: tools

- **CDFL:** Program analyser for proving the absence of runtime errors in program with floating-point computations based on Conflict-Driven Learning

- **CBMC:** state of art bounded mode checkers

- **CPBPV_FP:** our constraint-based bounded- model checking framework

Experiments: Program Heron

Uses Heron's formula to compute the area of a triangle from the lengths of a, b, and c (a being the longest side):

area= sqrt(s*(s-a)*(s-b)*(s-c) with **s=(a+b+c)/2**

```
/* Pre-condition :  a ≥ b and a ≥ c */
float heron(float a, float b, float c) {
  float s, squared_area;

  squared_area = 0.0f;
  if (a <= b + c) {
    s = (a + b + c) / 2.0f;
    squared_area = s*(s-a)*(s-b)*(s-c);
  }
  return sqrt(squared_area);
}
```

Optimized Heron : squared_area = ((a+(b+c))*(c−(a−b))*(c+(a−b))*(a+(b−c)))/16.0f;

Experiments

Name	Condition	CDFL	CBMC	std	fpc	fpc3s	s?
heron	aera < 10_f^{-5} area > 156.25f +10_f^{-5}	3.87 s > 180 s	0.28 s 34.51 s	> 180 s 22. 32 s	0.70 s 7.80 s	0.02 s n 0.08 s n	y y
optimized heron	aera < 10_f^{-5} area > 156.25f +10_f^{-5}	7.61 s > 180 s	0.93 s > 180 s	> 180 s 8.99 s	0.15 s 30.48 s	0.01 s n 0.01 s n	y n

std: standard *prune & bisection-based search*

fpc: domain of selected variable is split in *5 intervals*

- *3 degenerated intervals*: the smallest float ***l***, the largest float ***r***, and the mid-point ***m***
- intervals ***]l, m[*** and ***]m, r[***

fp3s: domain of selected variable is split in 3 degenerated intervals: the smallest float ***l***, the largest float ***r***, and the mid-point ***m***

Fault localization

- **Problem:**
  - Execution trace: often *lengthy* and *difficult* to understand
  - *Difficult to locate* the faulty statements

- **Goal:** Provide helpful information for *error localization on numeric constraint systems*
- **Input:**
  - Some imperative program with *numeric statements* (over integers or floating-point numbers)
  - *An assertion* to be checked
  - A *counter-example* that violates the assertion
- **Output**: information on locations of *potentially faulty statements*

Fault localization – Keys issues

- **What paths to analyse?**
  - Path from the counterexample
  - ***Deviations*** from the path from the counterexample

- **How to identify the suspicious program statements**
  - Computing *Maximal sets of statements satisfying the postcondition* → ***M****aximal* ***S****atisfiable* ***S****ubset*
  - *Computing Minimal sets of statements to withdraw* → ***M****inimal* ***C****orrection* ***S****et ?*

MSS, MCS: Definitions

- **MSS** Maximal Satisfiable Subset

  a generalization of MaxSAT considering maximality instead of maximum cardinality

  $M \subseteq C$ is a MSS $\Leftrightarrow$ M is SAT and $\forall c_i \in C \setminus M : M \cup \{c_i\}$ is UNSAT

- **MCS** Minimal Correction Set

  the complement of some MSS: removal yields a satisfiable MSS (it "corrects" the infeasibility)

  $M \subseteq C$ is a MCS $\Leftrightarrow$ $C \setminus M$ is SAT and $\forall c_i \in M : (C \setminus M) \cup \{c_i\}$ is UNSAT

LocFaults – Selecting Diverted Paths

- **Explore the path of the counter-example** and ***paths with at most** k* ***deviations***

  Example: one deviation

  Decision for one conditional statement is switched and the input data of the counter-example are propagated → new path **P'**

  Iff $CSP_{P'} \cup CSP_{POST}$ ***is satisfiable, MCS are computed for P'***

- **Compute MCS** with ***at most m suspicious statements***

  Bounds k and m are mandatory because there are an exponential number

  of paths and sets of suspicious statements

LocFaults – Computing MCCs for Diverted Paths

Let be:

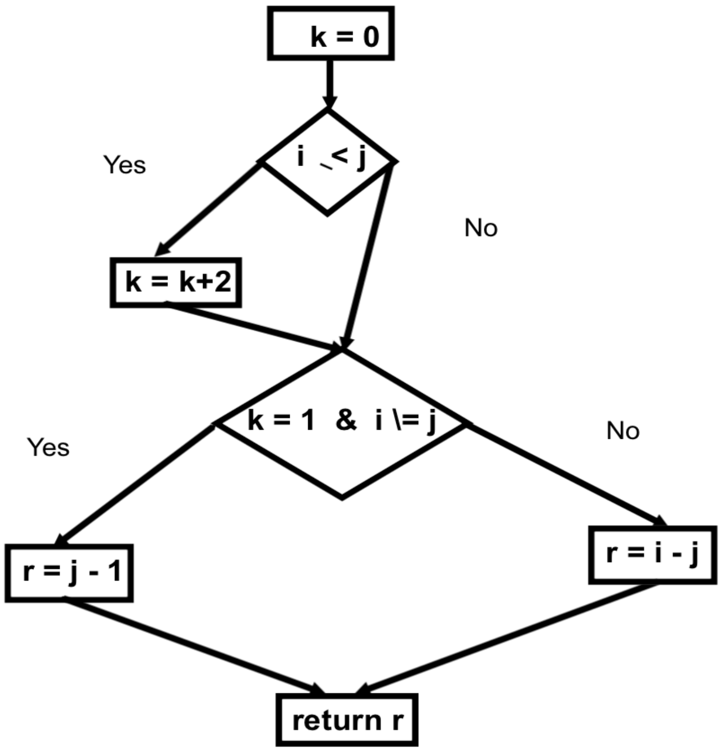
- **P**, a path generated by **k decision switches** of conditional statements **$cond_1$, .., $cond_k$** and by the propagation of **CE**
- **C**, the constraints of **P**, and **C_k**, the constraints generated by the assignments **occurring before $cond_k$** along **P_k**

If **C ∪ POST holds**:

- **{¬$cond_1$, .., ¬$cond_k$ }** is **a potential correction,**
- The **MCS** of **C_k ∪ {¬$cond_1$, .., ¬$cond_k$ }** are **potential corrections**

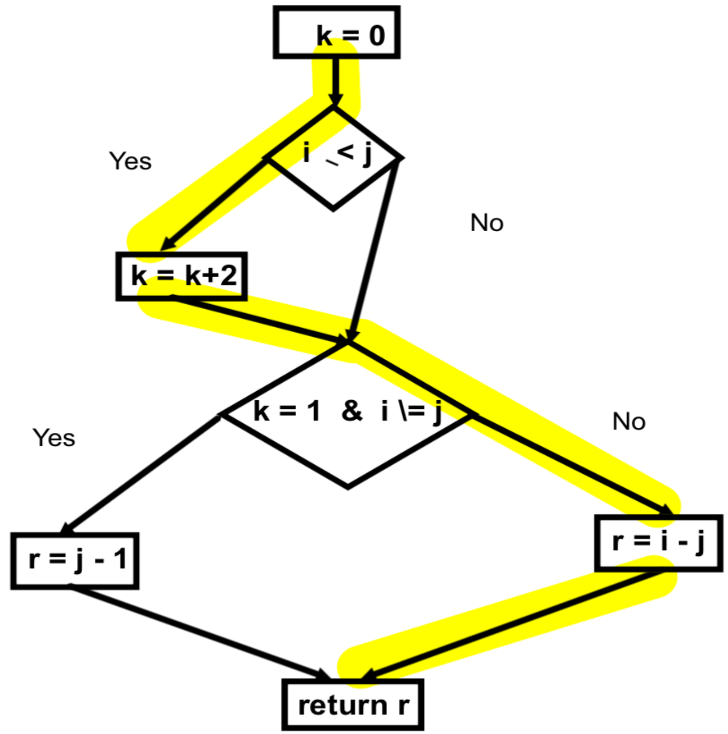
Note: {¬*cond*$_1$, .., ¬*cond*$_k$ } is a ”hard” constraint

LocFaults – Exemple



CFG of AbsMinus

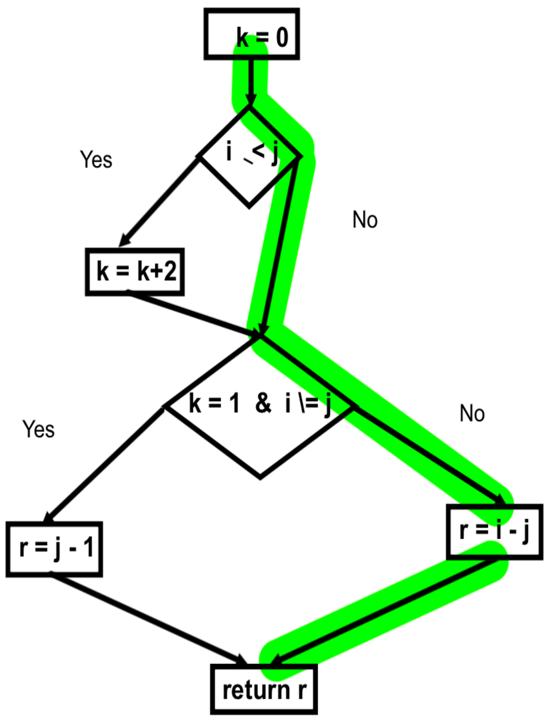
CE: {i =0,j =1}

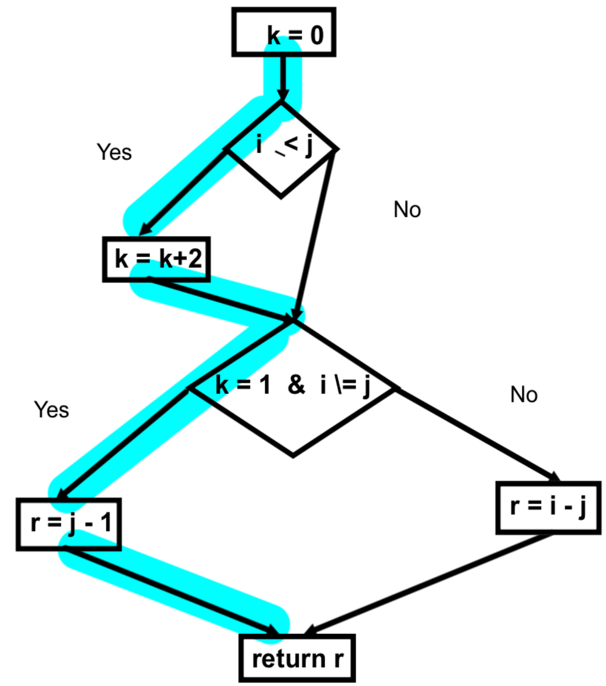


Faulty path for {i = 0, j = 1}

→ **Suspicious statement: {r= i - j}**

LocFaults – Exemple (cont.)





Change decision for 1st **IF**
Post-condition is **violated**
→ Path diversion **Rejected**

Change decision for 2d **IF**: Post-condition **holds**

CSP: $k0 = 0 \wedge k1 = k0 + 2 \wedge \neg((k1 = 1 \& I \neq j))$

Potential corrections: {k0 =0},{k=k+2},
{k=1&I ≠ j}

Computing all MCS(Minimal Correction Set)

Liffiton & Sakallah-2007

All_MCSes(ϕ)

1. $\phi' \leftarrow$ AddYVars(ϕ) *% Adds y_i selector variables*
2. MCSes $\leftarrow \varnothing$
3. k $\leftarrow$ 1
4. **while** (SAT(ϕ'))
5. $\phi'_k \leftarrow \phi' \wedge$ **AtMost**($\{\neg y_1, \neg y_2, \ldots, \neg y_n\}$, k)
6. **while** (newMCS $\leftarrow$ IncrementalSAT(ϕ'_k)) *%All MCS of size K*
7. MCSes $\leftarrow$ MCSes $\cup$ {newMCS}
8. $\phi'_k \leftarrow \phi'_k \wedge$ **BlockingClause**(newMCS) % Excludes super sets for *for size= k*
9. $\phi' \leftarrow \phi' \wedge$ **BlockingClause**(newMCS) *% Excludes super set for size > k*
10. **end while**
11. k$\leftarrow$k+1
12. **end while**
13. **return MCSes**

- *Incremental solver (MiniSAT) can be used in loop (l. 6) because constraints are only added but not external loop(l.4) since incrementing k relaxes constraints*
- *The set of yi variables assigned to false indicates the clauses in MCS*

LocFaults – experiments

Benchs	CE	E	Locfaults				BugAssist	
			0		1			
V7	i=2,j=1, k=2	58	**58**	0,77 s	{31},{37}, {27},{32}	**0,86 s**	{72, 37, 53, 49, 29, 35, 32, 31, 28, 65, 34, 62}	**20,48 s**
V8	i=3,j=4, k=3	61	**61**	0,74 s	{29},{35}, {30},{25}	**0,88 s**	{19, **61**, 79, 35, 27, 33, 30, 42, 29, 26, 71, 32, 48, 51, 44}	**25,72 s**

BugAssist: global approach based on MaxSat, merges the complements of ***MaxSat*** in a single set of suspicious statements

V7 and **V8** : variations of ***Tritype***

Input: three ***positive integers***, the triangle sides

Output: type of triangle (isosceles, equilateral, scalene, not a triangle)

V7 returns the ***product of the 3 sides***

V8 computes **the *square of the surface of the triangle by using Heron's formula***

LocFaults – Sum up

- **Flow-based** and **incremental** approach
  $\longrightarrow$ locates ***suspicious statements around the path*** of the counter-example

- **Constraint-based framework**
  - well adapted for handling arithmetic operations … on ***integers***
  - can be extended for handling programs with ***floating-point numbers*** computations (**?**)

Conclusion

- **BMC (Bounded Model Checking)**
  - Goal : ***Finding counter-examples violating an assertion***
  - Contribution of CP: ***Various solvers*** and ***search strategies***
  - Limit of CP: ***efficient pruning is a critical issue***

- **Program analysis**
  - Goal: ***Get rid of false alarms***
  - Contribution of CP : ***Refining abstraction, suspicious values***
  - Limit of CP: ***high computation cost***

- **Fault localization**
  - Goal: ***locations of potentially faulty statements***
  - Contribution of CP : **flow-based & incremental approach**
  - Limit of CP: **no global view**