ON THE CAPABILITIES OF CP FOR NUMERICAL PROGRAM ANALYSIS

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Outline

• **BMC** (Bounded Model Checking)
  - Goal: *Finding counter-examples violating an assertion*
  - State of the art Methods $\rightarrow$ *SAT /SMT Solvers*

• **Program analysis**
  - Goal: *Get rid of false alarms*
  - State of the art Methods $\rightarrow$ *abstract interpretation*

• **Fault localization**
  - Goal: *locations of potentially faulty statements*
  - State of the art Methods $\rightarrow$ *MaxSat*
Bounded Model Checking

- **Context**: programs with *numeric operations* over integer or floating point numbers

- **Goal**: *Finding counter-examples violating an assertion*
Bounded Model Checking framework

**Models** → finite automates, labelled transition systems

**Properties:**
- **Safety** → something bad should not happen
- **Liveness** → something good should happen

**Bound** $k$ → look only for counter examples made of $k$ states
Bounded Model Checking framework (cont.)

% set of states: S, initial states: I, transition relation: T

% bad states B reachable from I via T?

bounded_model_checkerforward(I,T,B,k)
  
  SC = ∅;  SN = I;  n=1
  
  while SC ≠ SN and n<k do
    
    If B ∩ SN ≠ ∅
    
    then return “found error trace to bad states”;
    
    else  S_C = S_N;  S_N = S_C ∪ T(SC);n = n + 1;
  
  done

return “no bad state reachable”;
SAT/SMT - Based BMC framework

1. The program is unwound $k$ times
2. The unwound (and simplified) program and the negation of the property are translated into a big propositional formula $\varphi$

   $\varphi$ is satisfiable iff there exists a counterexample of depth less than $k$

**SAT solvers** solvers have a “Global view”

- Numerical expressions $\rightarrow$ Boolean abstraction
- $\rightarrow$ Spurious solutions

**Critical issue:** relevant minimum conflict sets to limit backtracks
CP-Based BMC framework

1. The program is unwound \( k \) times
2. The unwound (and simplified) program in \( \text{SSA/DSA form} \) and the negation property are translated on the fly into constraint system \( C_s \)

\( C_s \) is satisfiable for a full path iff there exists a counterexample of depth less than \( k \)

Various solvers and strategies can be used

To explore only a limited part of the search space, efficient pruning is a critical issue
CP-Based BMC: CPBPV, a depth first strategy

CPBPV:

• Translate *precondition* (if exists) and *property* to check into a set of constraints

• *Explore each branch Bi* of the program and translate statements of branch *Bi* into a set of constraints
  
  o If for each branch *Bi*, the generated *CSP is inconsistent*, then the *program is conform* with its specification
  
  o If for some branch *Bi* the generated *CSP has a solution*, then this solution is a *counterexample* → exhibits a *non-conformity*

*Inconsistencies are detected at each node of the control flow graph*
CP-Based BMC: DPVS, a Dynamic Backjumping Strategy

Start from the post-condition and jump to the first locations where the variables of the post-condition are assigned

Essential observation:
When the program is in an SSA-like form, CFG does not have to be explored in a top down (or bottom up) way

→ compatible blocks can just be collected in a non-deterministic way

Why does it pay off?
- Enforces the constraints on the domains of the selected variables
- Detects inconsistencies earlier
To prove property $p_1$, select node (12), then select node (4)

→ the condition in node (0) must be true

$S = \{c_1 < d_0 + e_0 \land c_1 = c_0 + d_0 + e_0 \land c_0 = a_0 \land a_0 \geq 0\} = \{a_0 < 0 \land a_0 \geq 0\}$ ... inconsistent

```c
void foo(int a, int b)
int c, d, e, f;
if(a>=0) {
    if (a<10) {f=b-1;}
    else {f=b-a;}
    c=a;
    if (b>=0) {d=a; e=b}
    else {d=a; e=-b;}
}
else {
    c=b; d=1; e=-a;
    if (a>b) {f=b+e+a;}
    else {f=e*a-b;}
}
c = c+ d + e;
assert(c>=d+e); // property $p_1$
assert(f>=-b*e); // property $p_2$
```
Select node (8) → condition in node (0) must be false:

\[ S = \{ c1 < d_0 + e_0 \land c1 = c_0 + d_0 + e_0 \land c_0 = b_0 \land a_0 < 0 \land d_0 = 1 \land e_0 = -a_0 \} = \{ a_0 < 0 \land b_0 < 0 \} \]

Solution \{ a0 = -1, b0 = -1 \}
CP-Based BMC: Static versus Dynamic Strategies

Two benchmarks:
- **Flasher Manager**, industrial application
- **Binary Search**

<table>
<thead>
<tr>
<th>Bench</th>
<th>DPVS</th>
<th>CPBPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>FM 5</td>
<td>0.5</td>
<td>1.24</td>
</tr>
<tr>
<td>FM 100</td>
<td>15.95</td>
<td>&gt; 600</td>
</tr>
<tr>
<td>FM 200</td>
<td>22.65</td>
<td>&gt; 600</td>
</tr>
<tr>
<td>BS 8</td>
<td>35</td>
<td>0.2</td>
</tr>
<tr>
<td>BS 16</td>
<td>&gt; 600</td>
<td>1.14</td>
</tr>
</tbody>
</table>

→ *Pruning is a critical issue*
CP-Based program analysis

• Context:
  – **Embedded Systems** (Anti-lock Braking System controller, ...) rely more and more on floating-point computations
  – **C language** is widely used for such applications (often C code generated from a Simulink model)
    
    **Floats** → a source of **execution errors**

• **Goal:** *Get rid of false alarms (generated by abstract interpretation tools)*
Problems with floating-point numbers

Rounding: *Counter-intuitive properties*
  - Arithmetic operators are neither associative nor distributive
  - Reasoning with *absorption* and *cancellation*

**Examples** (in simple precision, binary representation):
  - Absorption: $10^7 + 0.5 = 10^7$
  - Cancellation: $((1 - 10^{-7}) - 1) \times 10^7 = -1.192... \neq -1$
  - $(10000001 - 10^7) + 0.5 \neq 10000001 - (10^7 + 0.5)$
  - $0.1 = (0.000110011001100...)$
Problems with floating-point numbers (cont.)

Programs are run on the floats but:

- **Specification, properties** of programs
  - Users are reasoning with real numbers
- **Programs** are often written with the semantics of real numbers “in mind”
- **Differences** between computations over real numbers and computations over the floats
  - Execution problems on programs with floats
Abstract Interpretation

Goal: static detection of execution errors

→ Approximations of computations over floats and of computations over the real numbers

Intervals, zonotopes, polyhedra...

Zonotopes: convex polytopes with a central symmetry (sets of affine forms)

+ Good trade-off between performance and precision

– Not very accurate for nonlinear expressions and on very common program constructs such as conditionals
Limits of Abstract Interpretation: false alarms

Possible trajectories

Forbidden zone
False alarm
Imprecise trajectory abstraction

Possible trajectories

Courtesy to Patrick Cousot
Abstract Interpretation: *good scalability* for estimating rounding errors but *over-approximation*

- *false alarms*
- totally *inappropriate behaviours* of a program may be dreaded but the developer does not know whether these behaviours will actually occur!

Constraint Programming:

*Good precision* (strong refutation capabilities, finding counter examples) but *lack of scalability*
Successive exploration and merging steps

- Use of *AI to compute a first approximation* of the values of variables at a program node where two branches join

- Building a constraint system for each branch between two join nodes in the CFG of the program and use of *CP local consistencies to shrink the domains* computed by AI
rAICP: example

On floats and reals, foo → z = [0,50]

Fluctuat → z = [0,100]

Merge points of foo: lines 13 and 21

lines 1 → 13:
Fluctuat → f, g, y ∈ [−10,10], x ∈ [−10, 0]

FPCS (path 1, “then” branch):
C = {x = f + 2*g ∧ x ≤ 0 ∧ y = g ∧ −10 ≤ f ∧ f ≤ 10 ∧ −10 ≤ g ∧ g ≤ 10 ∧ −10 ≤ y ∧ y ≤ 10 ∧ −10 ≤ x ∧ x ≤ 0}
→ g, y ∈ [−10,5]

lines 14 → 22:
Fluctuat → z ∈ [0,50]

1 /* Pre-condition:  f, g ∈ [−10,10] */
2 float foo(float f, float g) {
3    float x, y, z;
4    x = f + 2 * g;
5    if (x <= 0) {
6        y = g;
7    }
8    else {
9        y = −g;
10    }
11    if (y >= 0) {
12        z = 10*y;
13    }
14    else {
15        z = −y;
16    }
17    return z;
18}
rAICP: Filtering techniques

FPCS: *solver over floating-point constraints* combining *interval propagation* with explicit *search*

- **Correct** solver over the floats: *no solutions are lost*
- Based on *2B-consistency* and *3B-consistency*

**Projection functions for floats:**

- *Direct projections*: straightforward adaptation of interval arithmetic
- *Inverse projections*: less intuitive, more complex (e.g., might need a larger format than the system variables)

- **Handling of rounding modes**, nonlinear expressions and the usual mathematical functions (trigonometric, . . .)
Experiments: eliminating false alarms

**CDFL:** Program analyser for proving the absence of runtime errors in program with floating-point computations based on *Conflict-Driven Learning*

<table>
<thead>
<tr>
<th></th>
<th>rAICP</th>
<th>Fluctuat</th>
<th>CDFL</th>
</tr>
</thead>
<tbody>
<tr>
<td>False alarms</td>
<td>0</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>Total time</td>
<td>40.55s</td>
<td>18.33 s</td>
<td>208.99 s</td>
</tr>
</tbody>
</table>

Computed on the 55 benches from CDFL paper (TACAS’12, D’Silva, Leopold Haller, Daniel Kroening, Michael Tautschnig)
Generating Test Cases inside Suspicious Intervals

- **Suspicious intervals** for $x$: $[x_F, x_R - \epsilon]$ or $[x_R + \epsilon, x_F]$

- Tolerance specified by the user: $\epsilon$

- **Question**: Can the program hit a forbidden zone over the floating-point numbers?
Proposed approach : CPBPV_FP

“Forward” propagation

Computing the suspicious interval of $x$
→ approximate the domain of $x$ over the reals by
→ approximate domain of $x$ over the floats by $[x_F, x_F]$

“Backward “ propagation

Computing test-cases inside a suspicious interval of $x$
→ Solving a bounded-model checking problem with domain of $x$ restricted to $[x_F, x_R - \varepsilon]$ or $[x_R + \varepsilon, x_F]$
CPBPV_FP : CP based BMC for floats

Outputs:

• A test case
  → *P can produce* a suspicious value for *x*

• A proof that no test case exists
  → the suspicious interval *can be removed*

*Only the case if the loops in P cannot be unfolded beyond the bound k*

• An inconclusive answer
  → *P may produce* a suspicious value

  *no test case could be generated*
  *but the loops in P could be unfolded beyond the bound k*
FPCS Search Strategies

- **std**: standard *prune & bisection-based search*

- **fpc**: domain of selected variable is split in *5 intervals*
  - 3 degenerated intervals: the smallest float $l$, the largest float $r$, and the mid-point $m$
  - intervals $[l, m[$ and $]m, r[$

- **fp3s**: domain of selected variable is split in *3 degenerated intervals*: the smallest float $l$, the largest float $r$, and the mid-point $m$
Experiments: tools

- **CDFL**: Program analyser for proving the absence of runtime errors in program with floating-point computations based on Conflict-Driven Learning

- **CBMC**: state of art bounded mode checkers

- **CPBPV_FP**: our constraint-based bounded- model checking framework
Experiments: Program Heron

Uses Heron’s formula to compute the area of a triangle from the lengths of a, b, and c (a being the longest side):

\[
\text{area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{with} \quad s = \frac{(a+b+c)}{2}
\]

---

```c
/* Pre-condition : a ≥ b and a ≥ c */
float heron(float a, float b, float c) {
    float s, squared_area;
    squared_area = 0.0f;
    if (a <= b + c) {
        s = (a + b + c) / 2.0f;
        squared_area = s*(s-a)*(s-b)*(s-c);
    }
    return sqrt(squared_area);
}
```

**Optimized Heron**: squared_area = \((a+(b+c))(c-(a-b))\times(c+(a-b))(a+(b-c))\)/16.0f;
### Experiments

<table>
<thead>
<tr>
<th>Name</th>
<th>Condition</th>
<th>CDFL</th>
<th>CBMC</th>
<th>std</th>
<th>fpc</th>
<th>fpc3s</th>
<th>s?</th>
</tr>
</thead>
<tbody>
<tr>
<td>heron</td>
<td>( \text{area} &lt; 10_f^{-5} ) &lt;br&gt;( \text{area} &gt; 156.25f +10_f^{-5} )</td>
<td>3.87 s&gt; 180 s</td>
<td>0.28 s&gt; 34.51 s</td>
<td>&gt; 180 s&gt; 22.32 s</td>
<td>0.70 s&gt; 7.80 s</td>
<td>0.02 s ( \text{n} )&gt; 0.08 s ( \text{n} )</td>
<td>( \text{y} ) ( \text{y} )</td>
</tr>
<tr>
<td>optimized heron</td>
<td>( \text{area} &lt; 10_f^{-5} ) &lt;br&gt;( \text{area} &gt; 156.25f +10_f^{-5} )</td>
<td>7.61 s&gt; 180 s</td>
<td>0.93 s&gt; 180 s</td>
<td>&gt; 180 s&gt; 8.99 s</td>
<td>0.15 s&gt; 30.48 s</td>
<td>0.01 s ( \text{n} )&gt; 0.01 s ( \text{n} )</td>
<td>( \text{y} ) ( \text{n} )</td>
</tr>
</tbody>
</table>

**std**: standard *prune & bisection-based search*

**fpc**: domain of selected variable is split in *5 intervals*

- *3 degenerated intervals*: the smallest float \( l \), the largest float \( r \), and the mid-point \( m \)
- intervals \( ]l, m[ \) and \( ]m, r[ \)

**fp3s**: domain of selected variable is split in *3 degenerated intervals*: the smallest float \( l \), the largest float \( r \), and the mid-point \( m \)
Fault localization

• **Problem:**
  – Execution trace: often *lengthy* and *difficult* to understand
  – *Difficult to locate* the faulty statements

• **Goal:** Provide helpful information for *error localization on numeric constraint systems*

• **Input:**
  • Some imperative program with *numeric statements* (over integers or floating-point numbers)
  • An *assertion* to be checked
  • A *counter-example* that violates the assertion

• **Output:** information on locations of *potentially faulty statements*
Fault localization – Keys issues

• What paths to analyse?
  – Path from the counterexample
  – *Deviations* from the path from the counterexample

• How to identify the suspicious program statements
  – Computing *Maximal sets of statements satisfying the postcondition* → *Maximal Satisfiable Subset*
  – Computing *Minimal sets of statements to withdraw* → *Minimal Correction Set*?
MSS, MCS: Definitions

• **MSS** Maximal Satisfiable Subset
  
a generalization of MaxSAT considering maximality instead of maximum cardinality
  
  \[ M \subseteq C \text{ is a MSS } \iff M \text{ is SAT and } \forall c_i \in C \setminus M : M \cup \{c_i\} \text{ is UNSAT} \]

• **MCS** Minimal Correction Set
  
the complement of some MSS: removal yields a satisfiable MSS (it “corrects” the infeasibility)

  \[ M \subseteq C \text{ is a MCS } \iff C \setminus M \text{ is SAT and } \forall c_i \in M : (C \setminus M) \cup \{c_i\} \text{ is UNSAT} \]
LocFaults – Selecting Diverted Paths

• Explore the path of the counter-example and paths with at most $k$ deviations

Example: one deviation

Decision for one conditional statement is switched and the input data of the counter-example are propagated $\rightarrow$ new path $P'$

$Iff$ $CSP_P \cup CSP_{POST}$ is satisfiable, $MCS$ are computed for $P'$

• Compute $MCS$ with at most $m$ suspicious statements

Bounds $k$ and $m$ are mandatory because there are an exponential number

of paths and sets of suspicious statements
LocFaults – Computing MCCs for Diverted Paths

Let be:

- **P**, a path generated by **k decision switches** of conditional statements \( \text{cond}_1, \ldots, \text{cond}_k \) and by the propagation of **CE**
- **C**, the constraints of **P**, and **C_k**, the constraints generated by the assignments occurring before \( \text{cond}_k \) along \( P_k \)

If **C ∪ POST** holds:

- \( \{\neg \text{cond}_1, \ldots, \neg \text{cond}_k \} \) is a potential correction,
- The **MCS** of \( C_k ∪ \{\neg \text{cond}_1, \ldots, \neg \text{cond}_k \} \) are potential corrections

Note: \( \{\neg \text{cond}_1, \ldots, \neg \text{cond}_k \} \) is a ”hard” constraint
LocFaults – Exemple

CFG of AbsMinus

CE: \{i =0,j =1\}

Faulty path for \{i = 0, j = 1\}

→ Suspicious statement: \{r = i - j\}
Change decision for 1st IF:
Post-condition is **violated** → Path diversion **Rejected**

Change decision for 2d IF: Post-condition **holds**
CSP: \( k_0 = 0 \land k_1 = k_0 + 2 \land \neg((k_1 = 1 \land i \neq j)) \)
Potential corrections: \{k_0 = 0\}, \{k = k + 2\}, \{k = 1 \land i \neq j\}
Computing all MCS (Minimal Correction Set)  
Liffiton & Sakallah-2007

\textbf{All\_MCSes}(\phi)
1. \( \phi' \leftarrow \text{AddYVars}(\phi) \) \quad \% Adds \( y_i \) selector variables
2. \( \text{MCSes} \leftarrow \emptyset \)
3. \( k \leftarrow 1 \)
4. \textbf{while} (\text{SAT}(\phi'))
5. \( \phi'_k \leftarrow \phi' \land \text{AtMost}(\{\neg y_1, \neg y_2, \ldots, \neg y_n\}, k) \)
6. \textbf{while} (\text{newMCS} \leftarrow \text{IncrementalSAT}(\phi'_k)) \quad \% All MCS of size \( K \)
7. \( \text{MCSes} \leftarrow \text{MCSes} \cup \{\text{newMCS}\} \)
8. \( \phi'_k \leftarrow \phi'_k \land \text{BlockingClause}(\text{newMCS}) \) \quad \% Excludes super sets for size= \( k \)
9. \( \phi' \leftarrow \phi' \land \text{BlockingClause}(\text{newMCS}) \) \quad \% Excludes super set for size > \( k \)
10. \textbf{end while}
11. \( k \leftarrow k+1 \)
12. \textbf{end while}
13. \textbf{return} \text{MCSes}

\begin{itemize}
  \item \textit{Incremental solver (MiniSAT) can be used in loop (l. 6) because constraints are only added but not external loop (l.4) since incrementing \( k \) relaxes constraints}
  \item \textit{The set of \( y_i \) variables assigned to false indicates the clauses in MCS}
\end{itemize}
## LocFaults – experiments

<table>
<thead>
<tr>
<th>Benches</th>
<th>CE</th>
<th>E</th>
<th>Locfaults</th>
<th>BugAssist</th>
</tr>
</thead>
<tbody>
<tr>
<td>V7</td>
<td>i=2,j=1, k=2</td>
<td>58</td>
<td>58</td>
<td>{31},{37}, {27},{32}</td>
</tr>
<tr>
<td>V8</td>
<td>i=3,j=4, k=3</td>
<td>61</td>
<td>61</td>
<td>{29},{35}, {30},{25}</td>
</tr>
</tbody>
</table>

**BugAssist**: global approach based on MaxSat, merges the complements of MaxSat in a single set of suspicious statements

**V7 and V8**: variations of **Tritype**

**Input**: three positive integers, the triangle sides

**Output**: type of triangle (isosceles, equilateral, scalene, not a triangle)

**V7** returns the *product of the 3 sides*

**V8** computes the *square of the surface of the triangle by using Heron’s formula*
LocFaults – Sum up

• Flow-based and incremental approach
  → locates suspicious statements around the path of the counter-example

• Constraint-based framework
  – well adapted for handling arithmetic operations ... on integers
  – can be extended for handling programs with floating-point numbers computations (?)
Conclusion

• **BMC (Bounded Model Checking)**
  o Goal: *Finding counter-examples violating an assertion*
  o Contribution of CP: *Various solvers and search strategies*
  o Limit of CP: *efficient pruning is a critical issue*

• **Program analysis**
  o Goal: *Get rid of false alarms*
  o Contribution of CP: *Refining abstraction, suspicious values*
  o Limit of CP: *high computation cost*

• **Fault localization**
  o Goal: *locations of potentially faulty statements*
  o Contribution of CP: *flow-based & incremental approach*
  o Limit of CP: *no global view*